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VP FORCE RESOURCE ALLOCATION. (U)

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VP FORCE RESOURCE ALLOCATION

KFR 125-77

29 July 1977

Contract N00014-77-C-0286

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The Commander, Patrol Wings, Atlantic Fleet (CPWL) initiated a study to improve the methodology of allocating resources, flight hours in particular, to the units of his command. This document records the initial results of that study.

A review was made of CPWL's mission priorities, resource availability and current data relating resource allocation to mission performance. Using extant budget and personnel records, plus the VP Data Base, statistical analyses were done to determine the correlation between resource allocations and performance → over

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Indicators (PIs). A model was designed and documented to allocate resources on the basis of PI optimization using a "steepest ascent" algorithm. Sensitivity analyses were planned to aid in validation and subsequent refinements of the model.

The scope of the study did not include detailed statistical analyses of the VP Data Base PIs relative to resource allocations. Future work was outlined to complete these analyses to refine the model.

TABLE OF CONTENTS

INTRODUCTION

Background

The Commander, Patrol Wing, Atlantic Fleet (CPWL) is responsible for the training, material condition and operational effectiveness of the land based patrol aircraft of the Atlantic Fleet. Included in his responsibilities is the task of allocating resources for operations to the fleet squadrons in the command. Currently, CPWL has no comprehensive means of investigating the optimality of resource allocations to his squadrons. In general, without a model to more rigorously compare alternative allocation schedules relative to performance, resource allocations have been made according to subjective rules. And particularly, with no objective method to assist with flight hour allocation, CPWL has had difficulty validating the need for operating funds.

CPWL needs a tool to help allocate its resources, and thereby justify its resource allocations by clearly tying resource expenditures to performance. An optimally allocated budget for these resources cannot, by definition, be made more efficient, so a cut in budget means a decrease in performance. The allocation tool should relate to performance because the performance levels are defensible, where the budget alone is not.

Ketron was assigned a project to develop a tool to assist CPWL with its resource allocation tasks. A model has been developed which represents CPWL's mission performance in response to various allocation schedules of major resources. The major resources include money, flight hours, aircraft, personnel, sonobuoys, torpedoes, training simulators, and several others (see Appendix A for a complete list of resources represented in the model.) The model Ketron has developed will guide a shift from rules-of-thumb to more explicitly justified allocations based upon optimization of mission performance.

The model explicitly relates resource expenditures to performance. On the basis of these relations it determines an optimal allocation schedule using a straightforward decision tree, allocating to the unit(s) which will increase most

in performance. The model consists of three parts: an input processor for data preparation, a solution algorithm to determine the optimal allocation schedule, and an output processor to interpret the results and perform sensitivity analyses.

The model is now in a form ready for implementation, but it may not be in its final form. A fair amount of in-depth sensitivity analysis and refinement of the model as sample runs are made by those familiar with CPWL operations may be necessary to mold the allocation model into its final form.

This report describes the approach to the development of an allocation model and presents the results of the project including the initial program documentation.

Project Objectives

The two major project objectives were to:

- Develop a tool to aid optimal allocation of CPWL's resources, flight hours in particular. Within that objective, it is implied that an initial definition of an "objective function" that represents the output of CPWL's missions and tasks: i.e., *exactly what is being optimized*, would be formulated. We have chosen a combination of selected, already available performance indicators (PIs). The PIs are the quantitative measures generated in the mission performance analyses performed during the VP Data Base project.[1/] Use of these performance indicators in the allocations model is the first step in the creation of a full management system for operational resources.

- Run sensitivity analyses; first, to validate the model, and second as a follow-on effort on a regular basis to refine the model and improve the interpretation of results.

1/ VP Data Base Project, ONR Contract N00014-77-C-0145 Performed by Ketron, Inc. for CPWL.

Other implied objectives are:

- Built-in adaptability of the tool to changes in resource levels, required minimum and maximum allocation levels (constraints), and performance measurement criteria.

- Maximum conceptual and material simplicity of the model commensurate with its utility, flexibility, and realism.

Future Tasks

This project's goal was to construct the model for determining the most effective allocation of resources, (particularly flight hours) based upon the effect of the allocation on mission performance. That goal was attained, but the functions relating resource allocation to performance were only estimated. The next step in the development of the final resource management system should be a study of the effect of resources on operational performance. Such a future study should rely on the VP Data Base for performance measurements and continue the review of resource expenditure begun as the first task of this project.

Technical Approach

The project to develop and implement this model was divided into four major tasks. These tasks with a short statement of the work accomplished follows:

Task I: Review the missions and tasks of the patrol wings to determine the division of tasks within PATWINGSLANT squadrons.

It is essential to the evaluation of the model to continuously evaluate the worth of these tasks in relation to the value of the patrol wings effort as a whole. (See "Model Inputs" section, esp. mission vlaue coefficient). The current work with the VP Data Base project, and a limited subjective approach were used to provide "Performance Indicators".

Future evaluations of mission performance measures will be based upon three sources: (1) model results, including sensitivity analyses, (2) performance indicators quantified from the growing VP Data Base, and (3) a Delphi method of subjective evaluation. Though the subjective evaluations by CPWL personnel are often as accurate as rigorous mathematical derivations, they are not as defensible from criticism, therefore this third source will only be used when necessary to supplement the other two.

Task II: Develop allocation constraints and outline a constraint management system (CMS). The constraints on resource availability and their regression to performance (see "Model Inputs" section) are derived from existing data bases (e.g., RAINFORM reports, personnel records, cost record). Many constraints are estimated from physical hardware capability, such as aircraft, sonobuoys, torpedoes, etc. Others have been set by operational command requirements and measures related to flight safety. A CMS details procedures for continuing and improving the gathering of data relating to constraint formulation and the analysis of regression coefficients. In some cases of limited data or no adequately related PI for a given resource, subjective estimates are made of piecewise linear regression coefficients.

Task III: Develop a model. This is closely tied to Task II, defining data and constraints needed. A logical form of the constraints, the solution algorithm, and prerequisite analyses for an input processor are complete. Specific structures of the problem led us to choose a "steepest ascent" allocation model (see "Candidate Algorithms" section). The program is written and matched to computer hardware available.

Task IV: Perform sensitivity analyses and model validation. Uncertainities in data, PI's, and constraints were investigated, and estimates made of values of the factors in the model. Further effort is needed to experiment with the properties of non-linear relationships, and modify them as necessary to refine the model. The utility of PI regression coefficients should be critiqued with recommendations for change if necessary.

Overview of Tasks

In the early stages of this project, having completed a large portion of Tasks I and II, it became apparent that the previously suggested linear programming solution algorithm was inappropriate to both the data available and the problem's logical structure (see discussion in next section). Therefore, a different form for the basic algorithm was developed in the Task III effort. The constraints, CPWL tasks and the factors involved in the allocation process evaluated in Tasks I and II were restated to match the form of the new algorithm. Task IV is now underway. Refining the model from results of sensitivity studies is contingent upon program installation and training of future users, and more in-depth correlation (regression) studies of the data bases. Some data inputs to the program are in extant personnel data files, but must be dug out by hand and rearranged in a suitable form before the model can fully demonstrate its potential realism. This final state of implementation should be done at Brunswick.

The next section of this report reviews our reasons for rejecting linear programming as a CPWL allocation tool.

Candidate Algorithms for Allocation Model

The first and most obvious possible source for a solution algorithm is linear programming. This method is best used when there are distinct, well defined tradeoffs in utilization of resources, for example when we can define that the use of one torpedo and ten flight hours is equivalent in performance effect with, say, twenty sonobouys, ten flight hours, and five simulator hours.

However, even if such trade-offs were well defined (which they aren't), in fact, even if we knew which resources could be substituted for others in any proportion (which we don't, except in a very few cases), then the problem would still make little sense because different resources available to CPWL are often used for different missions. The results desired from completion of different missions, are definitely not translatable into one another.

Therefore, the tradeoff capability of linear programming is not applicable to this problem.

The same logic is true for the second most obvious algorithm source, non-linear programming. Added disadvantages of non-linear programming as a tool for an operating unit are the large computer requirements, intricate mathematical orientation, and often complicated computer programs.

Therefore, we discarded these classical programming allocation models and have used a steepest ascent allocation model instead. This solution algorithm is basically a straightforward decision tree. It starts with nothing allocated, then for each resource separately, it allocates incremental portions on the basis of gaining the maximum increase in performance. Piecewise linearities of regression coefficients are easily incorporated, unlike linear programming. For example, saturation effects cause the correlation or regression coefficients relating performance to the amount of resources to eventually decrease for large allocations, that is, in general the rate of increase of performance will decrease for very large amounts of any specific resource. The model is capable of representing this and other non-linearities in the relation between resources and performance.

For some exotic non-linearities, the steepest ascent model may not yield a true globally optimal solution. But for the fairly "smooth" curves realistically used in the model, this non-optimality will probably never occur.

The steepest ascent model is conceptually much simpler than linear programming. The computer program to run it is extremely simple, small, fast running, and easily debugged; it is even better than a simplex linear program in this respect. Its flexibility for refinement and future modification is easier and more intuitively logical than LP, and requires no mathematical expertise.

One might infer that the steepest ascent model would always be preferable to LP or non-linear programming, but this is not the case. They are not generally substitutable because they are used to solve different types of problems. It is the non-tradeoff structure of CPWL's allocation problem that allows the use of the steepest ascent solution algorithm.

MODEL DESCRIPTION

The model consists of three basic parts: an input processor, a solution algorithm, and an output processor. The input and output processors each have two functions: administrative and analytic. In both processors, the administrative function interacts with the user for input and output of data. The analytic function of the input processor prepares data inputs to the solution algorithm. The analytic function of the output processor interprets the output from the solution algorithm. The solution algorithm itself is the operating heart of the model. It calculates the optimal resource allocation schedule.

The solution algorithm is, of course, designed to solve a specific problem. The following sections discuss the structure of the problem, data requirements and formats, and the method of determining the optimal solution.

Variable Definitions

Appendix B lists definitions of variables used in the model. These are self-explanatory. For convenience, in some cases different symbols are used in the text than in the computer coding. Text notation and coding are compared in the appendix.

Current rough estimates of parameters are listed in Appendix C. These have been obtained from field sources. Some variables can be determined using Brunswick's data sources. Among these are numbers of personnel on board, which will come from extant personnel records; minimum flight hour requirements, which will come from average replacement rates and required hours for training and safety; detailed cost per flight or simulation hour for various missions; and reliable estimates of flight time lost due to aircraft being NORS and NORM.

Model Inputs

There are three types of explicit inputs to the solution algorithm and a fourth type used only by the output processor. The first is raw, directly usable data such as known available resources, constraint levels, and unit

costs. The second is input processor data, that is, data which must be analyzed and modified by the input processor before it can be used by the solution algorithm. This includes personnel numbers and qualifications which must be analyzed to forecast crew numbers and qualifications, which can then be used by the solution algorithm. The third is piecewise linear regression data which describes the effect of a given resource expenditure on mission performance. An example of a typical regression curve relating resource expenditure to performance is shown in Figure 1.

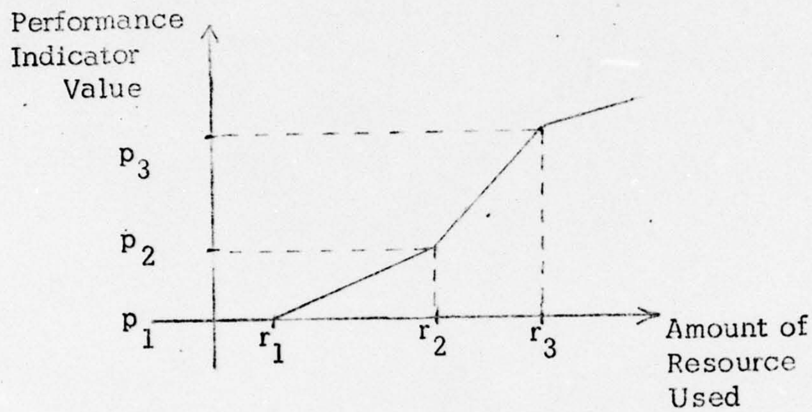

FIG. 1: TYPICAL RESOURCE REGRESSION CURVE

Input of these piecewise linear factors is done by entering ordered pair coordinates of the segment endpoints such as (r_1, p_1), (r_2, p_2), etc. The model will interpolate the rest of the curve. Whenever possible the regression curves are derived by statistical (regression) means. Otherwise the curves are estimated by subjective observation of mission performance and resource allocation.

Statistical analysis of RAINFORM reports, in particular the elapsed time data for the ASW missions, that is, on-station time, search time, localization time, tracking time, etc., should yield estimates of regression coefficients. Similar analyses can be done for most of the other missions.

The fourth explicit input is correlation data. Correlation is a statistical measure of factor dependence, and will be used extensively by the output processor in model validation and sensitivity analyses.

Implicit inputs to the model include data analysis functions for the input processor. These functions can be refined and modified fairly easily by a computer programmer if the initial work with the model indicates that further sophistication of the functions is needed. New functions can be added as well if, for instance, a new resource is considered for entry into the model.

Though much of the raw usable data, such as amounts available of each resource, is objective and easily quantified, at least one input, the mission value coefficient, is highly subjective. This coefficient portrays the relative importance of the various missions. It is the one means of discriminating between the net importance of performance in different missions. Therefore, it has a large impact on the optimal allocation of resources. Initially, its value is based on mission priorities as viewed by the commands of CPWL. Subsequent refinements of this factor will be based on sensitivity analyses.

Problem Structure

Appendix D lists the objective function (performance measure) to be maximized, and the constraints and consistency checks to which optimization is subject. The form and legends of the constraints and consistency checks are self-explanatory, but the structure of the objective function is less obvious, and will now be explained.

In essence, the objective function is a composite of many performance indicators multiplied and added together. The basic building blocks of the objective function are the PIs as given in Figure 1 of the previous section. We can summarize Figure 1 by equation (1).

$$PI_{ijkt} = f(R_{ijkt}) \tag{1}$$

That is, the performance of some portion of mission i relevant to resource k (by squadron j at time t) is equal to a function of the amount of resource k allocated to that mission (and squadron and time).

For consistency we have the normalizing condition

$$0 \leq PI_{ijkt} \leq 1 \tag{2}$$

Several resources may be used for each mission, so to calculate a *gross* PI for the whole mission, we multiply together the basic PIs for each resource.

$$\text{Gross Mission } PI_{ijt} = \prod_{k} PI_{ijkt}$$

where Π is the standard symbol for continued product. At this stage we multiply the PIs, instead of adding them, for two logical (though somewhat arbitrary) reasons: (1) We want the Gross Mission PI to be between zero and one. Summation would not ensure this. And more importantly (2) we believe the extra separability of addition should only apply to a mission-wide PI, and not to the separate parts of a single mission. This is consistent with the non-tradeoff aspect of the steepest ascent model, in that no small portionate PI of one mission can be separated out and exchanged for another small portionate PI of another mission.

There are some resources not explicitly allocated to a given mission which nonetheless affect mission performance. Among these "resources" are crew qualification levels, q, and training time available for each personnel category, l.

Thus a more accurate Gross Mission PI would be given by

$$\text{Gross Mission } PI_{ijt} = \prod_{k} PI_{ijkt} \prod_{l} PI_{ljt} \prod_{q} PI_{qjt} \tag{4}$$

Finally, if we are to meaningfully compare the PIs of different missions, we must take into account the relative mission priorities. We do so by weighting the Gross Mission PI with the mission value coefficient, E_{ijt}, (see Model Inputs section). This yields equation (5).

$$\text{Net Mission } PI_{ijt} = E_{ijt} \prod_{k} PI_{ijkt} \prod_{l} PI_{ljt} \prod_{q} PI_{qjt} \tag{5}$$

We now have comparable measures of mission performance, and can average them over all missions i, squadrons j, and time t, as shown in equations (6), (7), and (8), respectively, to yield a Total PI.

$$\text{Single-Squadron PI}_{jt} = \frac{1}{7} \sum_{i=1}^{7} \text{Net Mission PI}_{ijt} \qquad (6)$$

$$\text{All-Squadron PI}_{t} = \frac{1}{12} \sum_{j=1}^{12} \text{Single-Squadron PI}_{jt} \qquad (7)$$

$$\text{Total PI} = \frac{1}{12} \sum_{t=1}^{12} \text{All-Squadron PI}_{t} \qquad (8)$$

$$= \frac{1}{12} \sum_{t=1}^{12} \frac{1}{12} \sum_{j=1}^{12} \frac{1}{7} \sum_{i=1}^{7} E_{ijt} \prod_{k} PI_{ijkt} \prod_{l} PI_{ljt} \prod_{q} PI_{qjt}$$

This latter is the objective function we wish to maximize.

Piecewise Linear Data

In the objective function in Appendix D (reproduced in equation (9) below, you will note coefficients prefixed by the letters W and B. These are piecewise linear coefficients and will now be explained.

$$\text{Objective Function} = \frac{1}{12} \sum_{t} \frac{1}{12} \sum_{j} \frac{1}{7} \sum_{i} \Big[E_{ijt} \prod_{k} (WR_{ijkt}\, R_{ijkt} + BR_{ijkt}) \qquad (9)$$

$$\prod_{l} (WS_{ljt}\, S_{ljt} + BS_{ljt})$$

$$\prod_{q} (WN_{qjt}\, N_{qjt} +. BN_{qjt}) \Big]$$

Recall from Appendix B that WR is a single symbol, the prefix W means "regression slope coefficient of...", and WR_{ijkt} means "regression slope coefficient of Resource k allocated to squadron j, mission i, at time t". WR does not mean "W times R". The symbol W never stands alone.

Similarly BR_{ijkt} means "regression intercept coefficient of Resource k allocated to ... etc". B also is merely a prefix. As we shall show next, the regression slope and intercept coefficients are essential to calculating the value of the

PI for a given resource. The regression slope coefficient is also essential to calculating the steepest ascent of the objective function, as we will show in the solution algorithm section.

Recall Figure 1 and the method of inputing the coordinates (r_1, p_1), (r_2, p_2), etc. to define the piecewise linear relation between the amount of a resource used and its associated PI value.

The value of any particular regression slope coefficient will shift depending on how much relevant resource has been allocated. For example, refer to Figure 2 below for a typical piecewise linear regression plot.

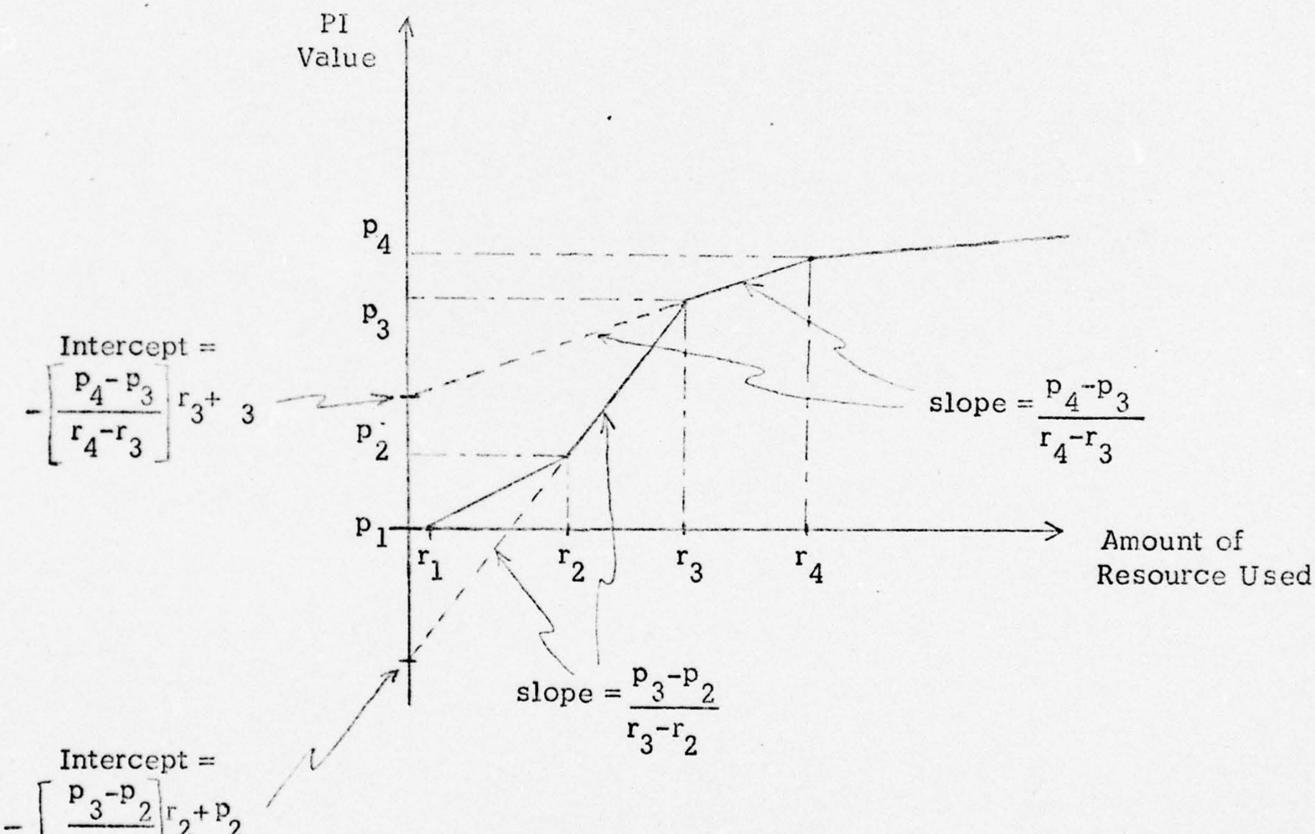

FIG. 2

If R_{ijkt} is the amount allocated, and $r_2 \le R_{ijkt} < r_3$, then the regression slope coefficient will be

$$WR_{ijkt} = \frac{p_3 - p_2}{r_3 - r_2} \tag{10}$$

and the regression intercept is given by

$$BR_{ijkt} = \left[\frac{p_3 - p_2}{r_3 - r_2}\right] (r_2) + p_2 \tag{11}$$

As the allocation progressively increases so that $r_3 \leq R_{ijkt} < r_4$, then the model automatically computes

$$WR_{ijkt} = \frac{p_4 - p_3}{r_4 - r_3} \tag{12}$$

and

$$BR_{ijkt} = \left[\frac{p_4 - p_3}{r_4 - r_3}\right] (r_3) + p_3 \tag{13}$$

as shown in Figure 2.

Thus the expression

$$PI_{ijkt} = f(R_{ijkt}) = WR_{ijkt} R_{ijkt} + BR_{ijkt} \tag{14}$$

is the value of the performance indicator for mission i, squadron j, at time t, when amount R_{ijkt} has been allocated.

Simulator hours (S) and number of qualified crews (N) are handled analogously. We have given them different symbols because they are iterated over different subscripts. As in equation (14), we have

$$PI_{ljt} = WS_{ljt} S_{ljt} + BS_{ljt} \tag{15}$$

and

$$PI_{qjt} = WN_{qjt} N_{qjt} + BN_{qjt} \tag{16}$$

Substituting (14), (15), and (16) into (8) yields (9), the objective function.

	1st tour	brand new	etc.....								
	Personnel Categories 1										
Crew Slot	1	2	3	4	5	6	7	8	9	10	11
1. Pilot			X	X	X						
2. CoPilot	X	X	X	X	X						
3. NAV	X	X									
4. COMM						X					
5. TACCO						X	X	X			
6. SS1										X	X
7. SS2									X	X	X
8. SS3									X	X	X
9. Arm.									X	X	X
10. ORD									X	X	X
11. FLT Eng									X	X	X
12. 2ndMech.									X		

FIG. 3

Solution Algorithm

The model's main solution algorithms are flow charted in Appendix E. Basically, all they do is allocate each resource, bit by bit, to wherever it will do the most good. They do this by calculating the steepest ascent of the objective function over all missions for a given resource, at a given squadron and time, and then allocating to that unit with the steepest ascent. There are some variations in handling subscripts for different resources because constraints have different forms, but basically they are all similar.

The ascent of the objective function is defined as its partial derivative with respect to the given resource. This is approximated by the mission value coefficient times the resource's regression slope coefficient, or

$$\text{ascent} = E_{ijt} \; WR_{ijkt}$$

In the flow charts this is abbreviated as E x W.

E x W is not the true partial derivative because the performance curves are in reality not independent of one another, that is, changing the allocation schedule of one resource would affect almost all the other performance curves of other resources. A simple example of this effect, using only two hypothetical resources, A and B, is illustrated in Figure 4.

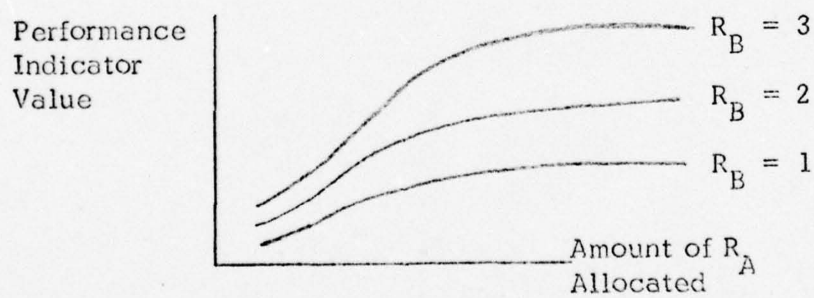

FIG. 4

The complexity of ten resources currently in the model would preclude explicitly defining these interrelations, even if data determinacy justified it, which it does not. But the model has two elements in its favor in this regard: (1) it implicitly

incorporates the interdependence, though only in a mathematically separable form, and (2) having many resources is actually an advantage, because the many fluctuations tend to cancel each other out, on the average, where only a few might not. Therefore we feel E x W is a justified approximation.

Simultaneous with this steepest ascent allocation, the solution algorithm keeps track of the non-linearities in the regression data, and ensures that constraints are not violated.

Output Processor

The output processor has two functions. Its administrative function is to arrange the output of the solution algorithm into a readable form. Its analytic function is to run sensitivity analyses and other calculations on this output.

In addition to the output of the solution algorithm (the optimal allocation schedule) the output processor requires two numbers for each set of regression coefficients: (1) the standard error of the estimate about the regression line, $S_{y.x}$, and (2) the standard error of the regression slope coefficient, S_o. These are easily calculated at the same time as the regression coefficients, before input to the model, using just a programmable pocket calculator. Computer time is not needed.

Estimates should be made of the expected error of subjectively (Delphi) determined variables. The subjective estimates of errors can conveniently be of the form, "We are around 90% certain that the estimate is correct to within about 20%." These estimates do not need to be precise to yield a meaningful sensitivity analysis.

Using these numbers, statistical confidence intervals for each resource can be calculated. These will indicate the confidence one can have that the performance measures in the model realistically correspond to real-world performance measures (assuming real-world allocations similar to those used in the model). This is the "model validation" subroutine.

In the "sensitivity analysis" subroutine we will investigate how the optimal solution is affected by various uncertainties in model inputs. For example, for a specific resource allocation, analysis of $S_{y.x}$ and S_o will yield a probability distribution for the values of relevant performance indicators. Rerunning the model in a Monte Carlo fashion for these probabilistic inputs will indicate sensitivity to allocation shifts.

Similarly, "shadow costs", that is, the effect on performance of increasing or decreasing available resource levels, can be quickly determined by looping back through particular parts of the model to recalculate the optimal solution for each of the various resource levels. The modular form of the model allows this to be done without rerunning the entire model.

APPENDIX A

Resources

DOLLARS FOR FLIGHT HOURS

DOLLARS FOR SIMULATOR HOURS

FLIGHT HOURS

SIMULATOR HOURS

AIRCRAFT

PERSONNEL

TORPEDOES

SONOBUOYS

HARPOONS

MINES

TRAINING AREAS

TRAINING RANGES

TRAINING SUBMARINES

APPENDIX B

Variables

Subscripts

i	=	task (1 to 7)
i'	=	ASW subtasks (1 to 6)
j	=	Squadron (1 to 12)
k	=	Resource (1 to 8)
l	=	Personnel category (1 to 11)
q	=	Crew qualification (1 to 4)
t	=	month (1 to 12)

Prefix

B	=	Regression intercept coefficient
C	=	Cost (money)
L	=	Lower bound
U	=	Upper bound
W	=	Regression slope coefficient

Suffix

T	=	Cumulative over time

i	=	Mission
1	=	Mobility
2	=	ASW
3	=	CAC
4	=	Recon
5	=	Mining
6	=	Elec. Surv.
7	=	Support

(Delete 8 = training, implicit in all others with S_{1jt} and N_{qjt}, see objective function.)

i'	=	ASW Operations
1	=	search
2	=	detect
3	=	transition
4	=	localization
5	=	track
6	=	kill

j	=	Squadron
1	=	VP-5
2	=	VP-8
3	=	VP-10
4	=	VP-11
5	=	VP-16
6	=	VP-23
7	=	VP-24
8	=	VP-26
9	=	VP-44
10	=	VP-45
11	=	VP-49
12	=	VP-56

k	=	Resource
1	=	flight hours
2	=	torpedoes
3	=	sonobouys
4	=	Harpoons
5	=	mines
6	=	training area
7	=	training range
8	=	submarines

q	=	Crew Qualification
1	=	C-1
2	=	C-2
3	=	C-3
4	=	C-4

resource	Relevant Missions i
flight hours	All
torpedoes	2
sonobouys	2
Harpoons	2
mines	5
training area	2-6
training range	2,5
submarines	2,3
simulator hours	1-6
crews	1-6

	1	=	Personnel Categories
PILOTS	1	=	1st tour brand new ($<$ 400 hours)
	2	=	1st tour with previous hours (400-700 hours)
	3	=	other 1st tour ($>$700 hours)
	4	=	2nd tour, not command level
	5	=	several tour command level
NFO's	6	=	1st tour
	7	=	2nd tour, not command level
	8	=	several tour command level
CREW MEMBERS	9	=	1st tour
	10	=	2nd tour usual crew members
	11	=	2nd tour, command billets

Definitions

AC_{jt}	=	number of aircraft (given)
CH_{ijt}	=	cost per flight hour (given)
CS_{jt}	=	cost per simulator hour (given)
$CHT_{quarter}$	=	dollars allocated to PATWINGS for flight hours (given)
$CST_{quarter}$	=	dollars allocated to PATWINGS for simulator hours (given)
E_{ijt}	=	mission value coefficient (given)
$E'_{i'jt}$	=	mission value coefficient for ASW subtasks (given, see "functions")
f (NORS, NORM)	=	fraction of available time which is free of maintenance on supply setbacks (given, see "functions")
$H_{ijt}(=R_{ij1t})$	=	allocation of flight hours (variable)
LE_{ijt}	=	required minimum effectiveness (given)
LHP_{ql}	=	minimum flight hours required per person (given)
LR_{ijkt}	=	minimum allocations required (given) $(K \neq 1)$
LS_{ljt}	=	minimum simulator hours required per person (given)
N_{qjt}	=	number of crews with C-q qualification (given, see "functions")
P_{qljt}	=	number of q-qualified, l-category personnel on board (given)
R_{ijkt}	=	amount of k^{th} resource allocated (variable)
$R_{ij1t}(=H_{ijt})$	=	allocation of flight hours (variable)
R_{ij2t}	=	torpedo allocations (variable)
R_{ij3t}	=	sonobouy allocations (variable)
R_{ij4t}	=	Harpoon allocations (variable)
R_{ij5t}	=	mine allocations (variable)
R_{ij6t}	=	flight hour allocations to training area (variable)
R_{ij7t}	=	flight hour allocations to training range (variable)

Definitions (contd)

R_{ij8t} = flight hour allocations to submarines (variable)

S_{ljt} = simulator hours allocated per man (variable)

UH_{jt} = upper bound on flight hours (given, see "functions")

UR_{ijkt} = maximum allocations allowed (given)

US_{ljt} = maximum simulator hours allowed per man (given)

UUS_t = overall maximum simulator hours available (given)

NOTE: The symbols listed as prefixes (B, C L U, and W) never stand alone as variables, they simply add a new meaning to whatever variable follows. For example, the variable R_{ijkt} denotes an allocation amount of resource k, and since the prefix W denotes "regression slope coefficient of ...", the compound symbol WR_{ijkt} means "regression slope coefficient of resource k when R_{ijkt} is the amount allocated." WR_{ijkt} does not mean W times R_{ijkt}. W has no *meaning by itself*, WR_{ijkt} is a single new name for a variable different from, but related to R_{ijkt}.

Similarly, the prefix U denotes, "upper bound of ...". Thus UR_{ijkt} means, the upper bound or R_{ijkt}". Obviously we must always have $R_{ijkt} \leq UR_{ijkt}$. Again, UR_{ijkt} is a single variable name. It does not mean U times R_{ijkt}.

Functions

$E'_{i'jt}$ = mission value coefficients for ASW subtasks (given)

Note: for ASW, $i = 2$

$$E_{2jt} = \sum_{i'} w'_{i'} E'_{i'jt}$$

where $0 \leq w'_{i'} \leq 1$ (given)

f(NORS, NORM) = .8

more complicated function may be used by the input processor

N_{qjt} = number of crews with C-q (eg. C-1, C-2, ...) qualification (given), currently optimally determined from personnel available by subroutine in the input processor.

UH_{jt} = upper bound on flight hours (given)

$$= \left(30 \frac{\text{days}}{\text{month}}\right) AC_{jt} \min \left\{ \frac{\left(24 \frac{\text{hours}}{\text{day}}\right) f(\text{NORS, NORM})}{(\text{sortie length}) + (\text{service time})}, \frac{\left(24 \frac{\text{hours}}{\text{day}}\right) \sum_{q} N_{qjt}}{(\text{sortie length}) + (\text{briefing time}) + (\text{rest time})} \right\}$$

airframe limit

crew limit

current assumptions:

sortie length	=	8 hours
service time	=	3 hours
briefing time	=	2 hours
rest time	=	8 hours

Computer Codes for Variables

Text Name	Computer Code Name	Dimensions	Total Words
AC_{jt}	AIRCFT (J)	(12)	12
CH_{ijt}	CFLTHR (I, base[1/])	(7, 6)	42
CS_{jt}	CSIMHR (base)	(6)	6
CHT quarter	CHT (quarter)	(4)	4
CST quarter	CST (quarter)	(4)	4
E_{ijt}	EFECT (I, base)	(7, 6)	42
$E'_{i'jt}$	EFECT 1 (I1, base)	(6, 6)	36
$H_{ijt} = R_{ij1t}$	FLTHR (I, J, T)	(7, 12, 12)	1008
N_{qjt}	QUAL (Q, J, T)	(4, 12, 12)	576
P_{qljt}	PERS (Q, L, J, T)	(4, 11, 12, 12)	6336
R_{ij2t}	TORP (J, T)	(12, 12)	144
R_{ij3t}	SONO (J, T)	(12, 12)	144
R_{ij4t}	HARP (J, T)	(12, 12)	144
R_{ij5t}	MINE (J, T)	(12, 12)	144
R_{ij6t}	AREA (J, T)	(12, 12)	144
R_{ij7t}	RNGE (I, J, T)	(2, 12, 12)	288
R_{ij8t}	SUBS (I, J, T)	(2, 12, 12)	288
S_{ljt}	SIMHR (L, J, T)	(11, 12, 12)	1584

[1/] base = IDT (J, T) = base where squadron j is stationed at time t, calculated by computer subroutine.

Text Name	Computer Code Name	Dimensions	Total Words
with prefixes:			
LE_{ijt}	LEFECT (I, base)	(7, 6)	42
LHP_{ql}	LHPERS (Q, L, T)	(4, 11, 15)	660
LR_{ij6t}	LAREA (base)	(6)	6
LR_{ij7t}	LRNGE (I, base)	(2, 6)	12
LR_{ij8t}	LSUBS (I, base)	(2, 6)	12
LS_{ljt}	LSIMHR (L, base)	(11, 6)	66
UH_{jt}	UFLTHR (base)	(6)	6
HR_{ij2t}	UTORP (base)	(6)	6
UR_{ij3t}	USONO (base)	(6)	6
UR_{ij4t}	UHARP (base)	(6)	6
UR_{ij5t}	UMINE (base)	(6)	6
UR_{ij6t}	UAREA (base)	(6)	6
UR_{ij7t}	URNGE (I, base)	(2, 6)	12
UR_{ij8t}	USUBS (I, base)	(2, 6)	12
US_{ljt}	USIMHR (L, base)	(11, 6)	66
UUS_{t}	UUSIM (T)	(12)	12
Correlation slope coefficients:			
WR_{ij1t}	WFLTHR (I, base, X, 2)	(7, 6, 5, 2)	420
WR_{ij2t}	WTORP (base, X, 2)	(6, 5, 2)	60
WR_{ij3t}	WSONO (base, X, 2)	(6, 5, 2)	60
WR_{ij4t}	WHARP (base, X, 2)	(6, 5, 2)	60
WR_{ij5t}	WMINE (base, X, 2)	(6, 5, 2)	60

Text Name	Computer Code Name	Dimensions	Total Words
WR_{ij6t}	WAREA (base, X, 2)	(6, 5, 2)	60
WR_{ij7t}	WRNGE (I, base, X, 2)	(2, 6, 5, 2)	120
WR_{ij8t}	WSUBS (I, base, X, 2)	(2, 6, 5, 2)	120
WS_{ljt}	WSIMHR (L, X, 2)	(11, 5, 2)	110
WN_{qjt}	WQUAL (Q, X, 2)	(4, 5, 2)	40

Allocation Increments	Unallocated Reserves	
DELFLT	RESFLT (T)	12
	QRSFLT (quarter)	4
DELTOR	RESTOR (T)	12
DELSON	RESSON (T)	12
DELHAR	RESHAR (T)	12
DELMIN	RESMIN (T)	12
DELARE	RESARE (T)	12
DELRNG	RESRNG (T)	12
DELSUB	RESSUB (T)	12
DELSIM	RESSIM (T)	12
	total words	13524

Miscellaneous

FLAG 1 indicates constraint #1 violation	
FLAGS (T) indicates simulator constraint violation	12

APPENDIX C

Estimates of Variable Values

(Forwarded Under Separate Cover)

APPENDIX D

Functional Forms

Objective Function

$$\text{Max} \ \frac{1}{12} \sum_t \frac{1}{12} \sum_j \frac{1}{7} \sum_i \left[E_{ijt} \prod_k \left(WR_{ijkt} R_{ijkt} + BR_{ijkt}\right) \prod_l \left(WS_{ljt} S_{ljt} + BS_{ljt}\right) \prod_q \left(WN_{qjt} N_{qjt} + BN_{qjt}\right) \right]$$

E_{ijt} = mission value coefficient

W prefix = regression slope coefficient

B prefix = regression intercept coefficient

R, S, N = amount of given resource allocated or available

Constraints, for each squadron, each month, site dependent

(1) $\frac{1}{12} \sum_{l} \sum_{q} LHP_{qlt} P_{qljt} \leq \sum_{i} H_{ijt}$

constrains flight hour allocation to meet minimum requirements over all personnel categories

(2) $\sum_{i} H_{ijt} \leq UH_{jt}$

sets maximum allowable flight hour allocation

(3) $LS_{ljt} \leq S_{ljt}$

constrains simulator hour allocation to meet minimum requirements per person

(4) $S_{ljt} \leq US_{ljt}$

sets maximum allowable simulator hour allocation per person

(5) $\sum_{t \in \text{quarter}} \sum_{j} \sum_{i} CH_{ijt} H_{ijt} \leq CHT_{\text{quarter}}$

constrains flight hour allocation within quarterly budget restriction

(6) $\sum_{t \in \text{quarter}} \sum_{j} \sum_{l} CS_{jt} S_{ljt} \left[\sum_{q} P_{qljt} \right] \leq CST_{\text{quarter}}$

constrains simulator hour allocation within quarterly budget restriction

(7)-(10) for $K = 2,3,4,5$: $R_{ijkt} \leq UR_{ijkt}$

sets maximum allowable allocations of: (7) torpedoes, (8) sonobuoys, (9) harpoons, and (10) mines

(11)-(13) for $K = 6,7,8$: $LR_{ijkt} \leq R_{ijkt}$

sets minimum required flight hour allocations to: (11) training areas, (12) training ranges, and (13) submarines

(14)-(16) for $K = 6,7,8$: $R_{ijkt} \leq UR_{ijkt}$

sets maximum allowed flight hour allocations to (14) training areas, (15) training ranges, and (16) submarines

(17) $\sum_{j} \sum_{l} S_{ljt} \left[\sum_{q} P_{qljt} \right] \leq UUS_{t}$

sets overall maximum available simulator hours

Consistency Checks (not Constraints)

(1) $$\frac{1}{12} \sum_{l} \sum_{q} LHP_{qlt} \; P_{qljk} \leq UH_{jt}$$

(2) $$LS_{ljt} \leq US_{ljt}$$

(3) $$\sum_{t \in \text{quarter}} \sum_{j} \sum_{l} \left[CS_{jt} \; LS_{ljt} \left[\sum_{q} P_{qljt} \right] \right] \leq CST_{\text{quarter}}$$

(4)-(6) for K = 6, 7, 8 : $LR_{ijkt} \leq UR_{ijkt}$

(7) $$LE_{ijt} \leq \left[E_{ijt} \prod_{k} (WR_{ijkt} \; UR_{ijkt} + BR_{ijkt}) \prod_{l} (WS_{ljt} \; US_{ljt} + BS_{ljt}) \prod_{q} (WN_{qjt} \; UN_{qjt} + BN_{qjt}) \right]$$

(Note: UN_{qjt} not specified, estimate = 5 for these purposes)

after allocation check:

(8) $$LE_{ijt} \leq \left[E_{ijt} \prod_{k} (WR_{ijkt} \; R_{ijkt} + BR_{ijkt}) \prod_{l} (WS_{ljt} \; S_{ljt} + BS_{ljt}) \prod_{q} (WN_{qjt} \; N_{qjt} + BN_{qjt}) \right]$$